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NEWTONIAN SNOWPLOW THEORY OF OSCILLATING AIRFOILS

J. Aroesty, A. F. Charwat, S. Y. Chen and J. D. Cole

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PREFACE

Unsteadiness effects upon the aerodynamic and stability characteristics of a slender body in hypersonic flight can be vitally important in determining reentry-vehicle and interceptor performance. In this Memorandum, a very simple physical model, Newtonian snowplow theory, is used to estimate the unsteady forces associated with the oscillatory motion about a fixed center of rotation of a slender two-dimensional airfoil which translates hypersonically through a uniform atmosphere.

SUMMARY

Unsteady forces acting on two-dimensional oscillating airfoils are calculated by Newtonian snowplow theory. A pressure formula which is valid for linear and nonlinear oscillations is obtained, and comparisons are made with the exact, linearized gasdynamic theory for a hypersonic slender wedge. These comparisons indicate that Newtonian theory is not very accurate for the calculation of unsteady stability derivatives and seems to be inferior to estimates made using simple impact theory.

CONTENTS

SYMBOLS

k	=	reduced frequency, $\omega\ell/U$, when α = real part of $\tilde{\alpha}_{max}e^{i\omega t}$
ℓ	=	reference length (length of airfoil); all lengths are nondimensionalized by ℓ
ℓ_1	=	nondimensional distance to center of rotation
M	=	pitching moment
m_α	=	aerodynamic stiffness, $\frac{1}{\rho_\infty U^2 \ell^2}\left(\frac{\partial M}{\partial \alpha}\right)_{\alpha=\dot{\alpha}=0}$
$m_{\dot{\alpha}}$	=	aerodynamic damping, $\frac{1}{\rho_\infty U \ell^3}\left(\frac{\partial M}{\partial \dot{\alpha}}\right)_{\alpha=\dot{\alpha}=0}$
p	=	pressure
t	=	time
U	=	velocity of translation
X	=	nondimensional distance in space-fixed coordinate system; $X = 0$ corresponds to location of nose at $t = 0$
X^*	=	nondimensional coordinate measured from nose in body-fixed coordinate system
Y^*	=	dimensionless body-surface ordinate
$\tilde{y}$	=	dimensionless transverse displacement of point on body surface
$\alpha(t)$	=	angular displacement
$\dot{\alpha}(t)$	=	angular velocity
$\ddot{\alpha}(t)$	=	angular acceleration
γ	=	isentropic exponent
ρ_∞	=	ambient density
τ	=	thickness parameter
ω	=	frequency of oscillation

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I. INTRODUCTION

Knowledge of the aerodynamic and stability characteristics of a slender hypersonic vehicle in unsteady flight is important for the accurate prediction of reentry-vehicle and interceptor performance. In general, the accurate determination of unsteadiness effects by the solution of time-dependent gasdynamic equations is quite complicated, and it is necessary to develop approximate techniques for estimating unsteady force coefficients. This Memorandum discusses the further application of a simple physical model, Newtonian snowplow theory, to the calculation of unsteady forces on an oscillating body.

The pressure acting on a two-dimensional or axisymmetric hypersonic missile undergoing rapid acceleration was estimated in Ref. 1, using Newtonian snowplow theory. The cross-plane analogy was introduced in order to relate the two-dimensional unsteady hypersonic motion of the missile surface to an equivalent unsteady one-dimensional piston motion, which was then solved by assuming a strong shock wave and the spatial uniformity of the gas between the piston surface and the shock wave.

The pressure on the piston surface was computed by the application of Newton's law in the form $P(t) = d/dt\,(m(t)v(t))$, where $m(t)$ is the mass of the gas in the shock layer between piston surface and shock wave, and $v(t)$ is the velocity of the gas in the shock layer, which is assumed to be identical with the piston velocity.

These same ideas are employed here to estimate the pressure on an oscillating two-dimensional slender body in hypersonic flight. A specific example, that of a thin wedge undergoing small-angle oscillations, is computed, and comparisons are then made with results of more accurate calculations using the equations of hypersonic gasdynamics, as well as with results of some wind-tunnel experiments.

These comparisons seem to indicate that Newtonian theory is not very accurate for the estimation of unsteady forces if $\gamma = 1.4$, probably because of the relatively large magnitude of the corrections due to the density ratio, $(\gamma - 1)/(\gamma + 1)$, being nonzero.

II. ANALYSIS

ARBITRARY SLENDER AIRFOIL

A two-dimensional airfoil traverses a uniform atmosphere with a constant hypersonic velocity of translation, U, and undergoes an oscillatory motion about the center of rotation, located at $X^* = \ell_1$, $Y^* = 0$.

All lengths are rendered nondimensional by the overall body length, ℓ. The X^*-coordinate system is fixed in the moving body and is measured from the nose, while the X-coordinate system is fixed in space and is measured from the horizontal location of the nose at time t = 0 (see Fig. 1).

The body shape is fixed and is described by

$$Y^* = \tau F(X^*) \tag{1}$$

where $\tau \ll 1$, and F(0) = 0, F(1) = 1.

For the present theory to apply, the angular displacement, $\alpha(t)$, is required to be no larger than τ, and the transverse velocity and acceleration induced by the oscillations are required to be no larger than those which occur over the same body in steady flight. Thus,

$$\alpha(t) = \tau\tilde{\alpha}(t) \tag{2}$$

where $\tilde{\alpha}(t) \sim 0(1)$; and

$$\dot{\alpha}(t) = \frac{\tau U}{\ell}\left(\frac{\ell}{U}\frac{d\tilde{\alpha}}{dt}\right) \tag{3}$$

where $\frac{\ell}{U}\frac{d\tilde{\alpha}}{dt} \sim 0(1)$; and

$$\ddot{\alpha}(t) = \frac{\tau U^2}{\ell^2}\left(\frac{\ell^2}{U^2}\frac{d\tilde{\alpha}^2}{dt^2}\right) \tag{4}$$

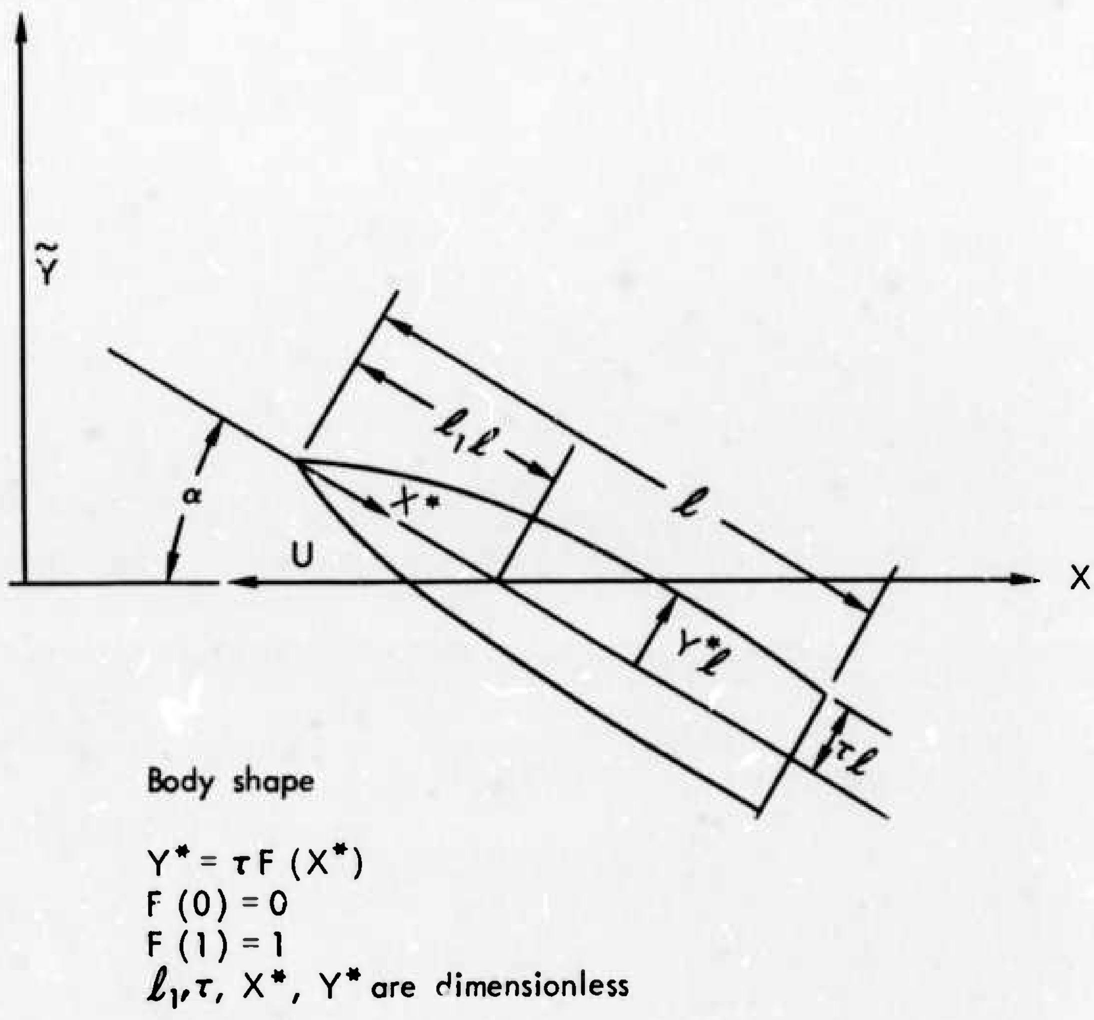

Fig. 1—Coordinate system

where

$$\frac{\ell^2}{U^2}\frac{d^2\tilde{\alpha}}{dt^2} \sim O(1)$$

With these orders of magnitude, the transverse displacement of a point on the upper surface measured from the horizontal, $\tilde{y}$, is given to terms of $O(\tau^2)$ by

$$\tilde{y} = \tau[F(X^*) + \tilde{\alpha}(\ell_1 - X^*)] \tag{5}$$

The dimensional transverse velocity of a point on the surface is

$$v \equiv \frac{d\tilde{y}}{dt}\cdot\ell = \ell\tau\left[\frac{dF}{dX^*}\cdot\frac{dX^*}{dt} + \frac{d\tilde{\alpha}}{dt}(\ell_1 - X^*) - \tilde{\alpha}\frac{U}{\ell}\right]$$

$$= \ell\tau\left[F_{X^*}\cdot\frac{U}{\ell} + \frac{U}{\ell}\left(\frac{\ell}{U}\frac{d\tilde{\alpha}}{dt}\right)(\ell_1 - X^*) - \tilde{\alpha}\frac{U}{\ell}\right] \tag{6}$$

and the dimensional transverse acceleration is then

$$\frac{d^2\tilde{y}}{dt^2}\cdot\ell = \ell\tau\left[\frac{1}{\ell^2}F_{X^*X^*}U^2 + \frac{U^2}{\ell^2}\left(\frac{\ell^2}{U^2}\frac{d^2\tilde{\alpha}}{dt^2}\right)(\ell_1 - X^*) - 2\frac{U^2}{\ell}\left(\frac{\ell}{U}\frac{d\tilde{\alpha}}{dt}\right)\right] \tag{7}$$

At the fixed station in space, $X = 0$, which corresponds to the X-coordinate of the nose at time $t = 0$, the flow pattern can be calculated as an unsteady one-dimensional flow with an equivalent piston velocity $d\tilde{y}/dt \cdot \ell$.

The mass of the gas (per unit piston area) in motion at time t at the fixed station $X = 0$ is approximated by the mass of all the gas which has been swept by the piston up to time t, or

$$m(t) = \rho_\infty\tau\left[\tilde{y}(t) - \tilde{y}_{nose}(t = 0)\right]\ell \tag{8}$$

Snowplow theory assumes that the velocity of the gas in the shock layer can be approximated by the velocity of the piston, so that the pressure on the piston, P(t), can be written

$$P = \frac{d}{dt}(mV) \tag{9}$$

where

$$V(t) = \ell \frac{d\tilde{y}}{dt}$$

or

$$\begin{aligned} P(X^*,t) = {} & \rho_\infty U^2 \tau^2 \left[F_{X^*} + \frac{\ell}{U}\frac{d\tilde{\alpha}}{dt}(\ell_1 - X^*) - \tilde{\alpha} \right]^2 \\ & + \rho_\infty U^2 \tau^2 \left[F(X^*) + \tilde{\alpha}(t)(\ell_1 - X^*) - \tilde{\alpha}(0)\ell_1 \right] \\ & \times \left[F_{X^*X^*} + \frac{\ell^2}{U^2}\frac{d^2\tilde{\alpha}}{dt^2}(\ell_1 - X^*) - 2\frac{\ell}{U}\frac{d\tilde{\alpha}}{dt} \right] \end{aligned} \tag{10}$$

Note that $\tilde{\alpha}(0) = \tilde{\alpha}[t - X^*(\ell/U)]$. If $\tilde{\alpha}(t)$ is the real part of $\tilde{\alpha}_{max} e^{i\omega t}$, then the pressure formula becomes

$$\begin{aligned} P(X^*,t) = {} & \rho_\infty U_\infty^2 \tau^2 \left[F_{X^*} + ik\tilde{\alpha}(\ell_1 - X^*) - \tilde{\alpha} \right]^2 \\ & + \rho_\infty U_\infty^2 \tau^2 \left[F_{(X^*)} + \tilde{\alpha}(\ell_1 - X^*) - \tilde{\alpha}\, e^{-ikX^*}\ell_1 \right] \\ & \times \left[F_{X^*X^*} - k^2(\ell_1 - X^*)\tilde{\alpha} - 2ik\tilde{\alpha} \right] \end{aligned} \tag{11}$$

where $k = \omega\ell/U$. This formula is not restricted to small values of the amplitude, $\tilde{\alpha}_{max}$; as long as the amplitude $\tilde{\alpha}_{max}$ is O(1), the pressure

formula should be applicable. This is not a linearized analysis of the unsteady motion, and it can be used to calculate force and stability coefficients for both large and small departures from a uniform hypersonic translation, as long as the slender-body approximation is still valid and the reduced frequency is not too large.

If the pressure, as calculated from Eq. (10), is greater than zero, then there should be no anomalies associated with the time-dependent detachment and reattachment of the Newtonian free layer. The present theory is not applicable to calculation of the unsteady oscillations of an axisymmetric body, since there is no equivalent unsteady one-dimensional piston motion.

If the amplitude, $\tilde{\alpha}_{max}$, is small, then Eq. (11) may be linearized in the angular displacement, which results in the following equation for the unsteady component of the pressure:

$$p_{unsteady} = \rho_\infty U^2 \tilde{\alpha} \tau^2 \Big\{ F_{X^*} 2[ik(\ell_1 - X^*) - 1] - F[k^2(\ell_1 - X^*) + 2ik] + F_{X^*X^*}[(\ell_1 - X^*) - e^{-ikX^*}\ell_1] \Big\} \tag{12}$$

EXAMPLE: SLENDER WEDGE

Restricting further to a slender wedge surface, where $F = X^*$, results in

$$\frac{p_{unsteady}}{\rho_\infty U^2 \tau^2 \tilde{\alpha}} = -2 - k^2(\ell_1 - X^*)X^* + 2ik(\ell_1 - 2X^*) \tag{13}$$

Following McIntosh,[2] Eq. (13) can be written in the form

$$C_{P_{lower}} - C_{P_{upper}} = 4\tau^2 \tilde{\alpha}\, e^{ikt}\left(L_1 + iL_2\right) \tag{14}$$

where the subscripts "lower" and "upper" refer to the respective wedge surfaces and L_1 and L_2 are real functions of X^* and k only, and where

$$L_1 = 2 + k^2(\ell_1 - X^*)X^* \tag{15a}$$

is the in-phase component of the pressure,

$$L_2 = -2k(\ell_1 - 2X^*) \tag{15b}$$

is the out-of-phase component of the pressure, and C_p, the pressure coefficient, is defined by $C_p = p/\rho_\infty (U_\infty^2/2)$.† When the appropriate Newtonian limiting process is applied to McIntosh's solution for the oscillating wedge in hypersonic flow, Eqs. (15a) and (15b) can be extracted.

McIntosh[(2)] has applied hypersonic small-disturbance theory to the calculation of the unsteady pressure component on a slender oscillating wedge. A linearization in the amplitude of the oscillation is employed, resulting in a theory which is valid for arbitrary γ, $M_\infty^2 \to \infty$, $\tau^2 \to 0$, $\tilde{\alpha} \to 0$, $k \sim O(1)$, and the full range of values of $M^2\tau^2$ between zero and infinity. Values of k which are of unit order are involved in flutter applications of unsteady-flow theory. For these values of k, McIntosh's solution is given as the sum of an infinite series, which has been computed for only a few cases.

McIntosh calculates the full solution for L_1 and L_2 for $\gamma = 1.4$, $k = 0.25$, and $\ell_1 = 0$ and 1 at certain values of X^*. An estimate of the accuracy of snowplow theory is possible by comparing with those results.

For $\ell_1 = 1$, $k = 0.25$, and $X^* = 2.0$, the exact hypersonic theory ($1/K^2 \to 0$, $\gamma = 1.4$) gives $L_{1_{exact}} = 2.02$, while the present theory gives $L_1 = 2.125$. For these same conditions, $L_{2_{exact}} = 0.85$, while the present theory gives $L_2 = 1.5$.

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†Impact Newtonian theory, obtained by neglecting all acceleration effects, yields $L_1 = 2$, $L_2 = -2k(\ell_1 - X^*)$.

For $\ell_1 = 0$, $k = 0.25$, $\gamma = 1.4$, and $1/K^2 \to 0$, $L_{1_{exact}} = 2.03$ and $L_{2_{exact}} = 0.35$. The present theory gives $L_1 = 1.97$ and $L_2 = 0.5$. At $X^* = 3$, under the same conditions, $L_{1_{exact}} = 2.85$ and $L_{2_{exact}} = 1.8$, while the present theory gives $L_1 = 2.58$ and $L_2 = 3$.

Application of unsteady-flow theory to dynamic-stability problems typically involves reduced frequencies, k, which are much less than one. For example, if $\omega = 10$ rad/sec, $\ell = 10$ ft, and $U = 10^4$ ft/sec, then $k = 10^{-2}$. For small values of k and $1/K^2 \to 0$, McIntosh's series formulae for L_1 and L_2 may be summed, resulting (for $\gamma = 1.4$) in

$$L_{1_{exact}} = 2.0$$

$$L_{2_{exact}} = 2.7\, kX^* - 2.0k\ell_1$$

which should be compared to Eqs. (15a) and (15b). McIntosh's calculations also indicate that due to $\epsilon = (\gamma - 1)/(\gamma + 1)$ being nonzero, the first correction term to Newtonian theory is not negligible at $\gamma = 1.4$ and has an especially large effect on the out-of-phase component, L_2.

The pitching moment about the center of rotation of a symmetric wedge is

$$\frac{M}{\ell^2} \equiv 2 \int_0 \tilde{P}(\ell_1 - X^*)\, dX^* \tag{16}$$

or, using Eq. (13),

$$M = -4\left(\ell_1 - \frac{1}{2}\right)\ell^2 \alpha\tau\rho_\infty U_\infty^2 + 4ik\left(\ell_1^2 - \frac{3\ell_1}{2} + \frac{2}{3}\right)\ell^2\alpha\tau\rho_\infty U_\infty^2 \tag{17}$$

Two dimensionless coefficients which have been measured on slender wedges in high-speed flow are the aerodynamic stiffness, defined by

$$m_\alpha = \frac{1}{\rho_\infty U_\infty^2 \ell^2}\left(\frac{\partial M}{\partial \alpha}\right)_{\alpha=\dot{\alpha}=0} \tag{18}$$

and the aerodynamic damping, defined by

$$m_{\dot{\alpha}} = \frac{1}{\rho_\infty U_\infty \ell^3}\left(\frac{\partial M}{\partial \dot{\alpha}}\right)_{\alpha=\dot{\alpha}=0} \tag{19}$$

The present theory gives

$$m_\alpha = -4\left(\ell_1 - \frac{1}{2}\right) \tag{20}$$

and

$$m_{\dot{\alpha}} = 4\left(\ell_1^2 - \frac{3\ell_1}{2} + \frac{2}{3}\right)\tau \tag{21}$$

Appleton(3) has calculated these coefficients, using a perturbation theory similar to that used by McIntosh, and has further made the low-frequency approximation $k \ll 1$. In addition, he reports the results of an experimental investigation of $m_{\dot{\alpha}}$, m_α performed by East(4) at Southampton on a slender wedge (wedge half-angle = 9.5 deg, M_∞ = 9.7) in air. (Here, Appleton's presentation of East's data is used.)

Appleton's theoretical results, East's experimental data, and the present results are plotted in Fig. 2. Also included are the results of Newtonian impact theory,

$$m_\alpha = 4\left(\ell_1 - \frac{1}{2}\right)\tau \tag{22}$$

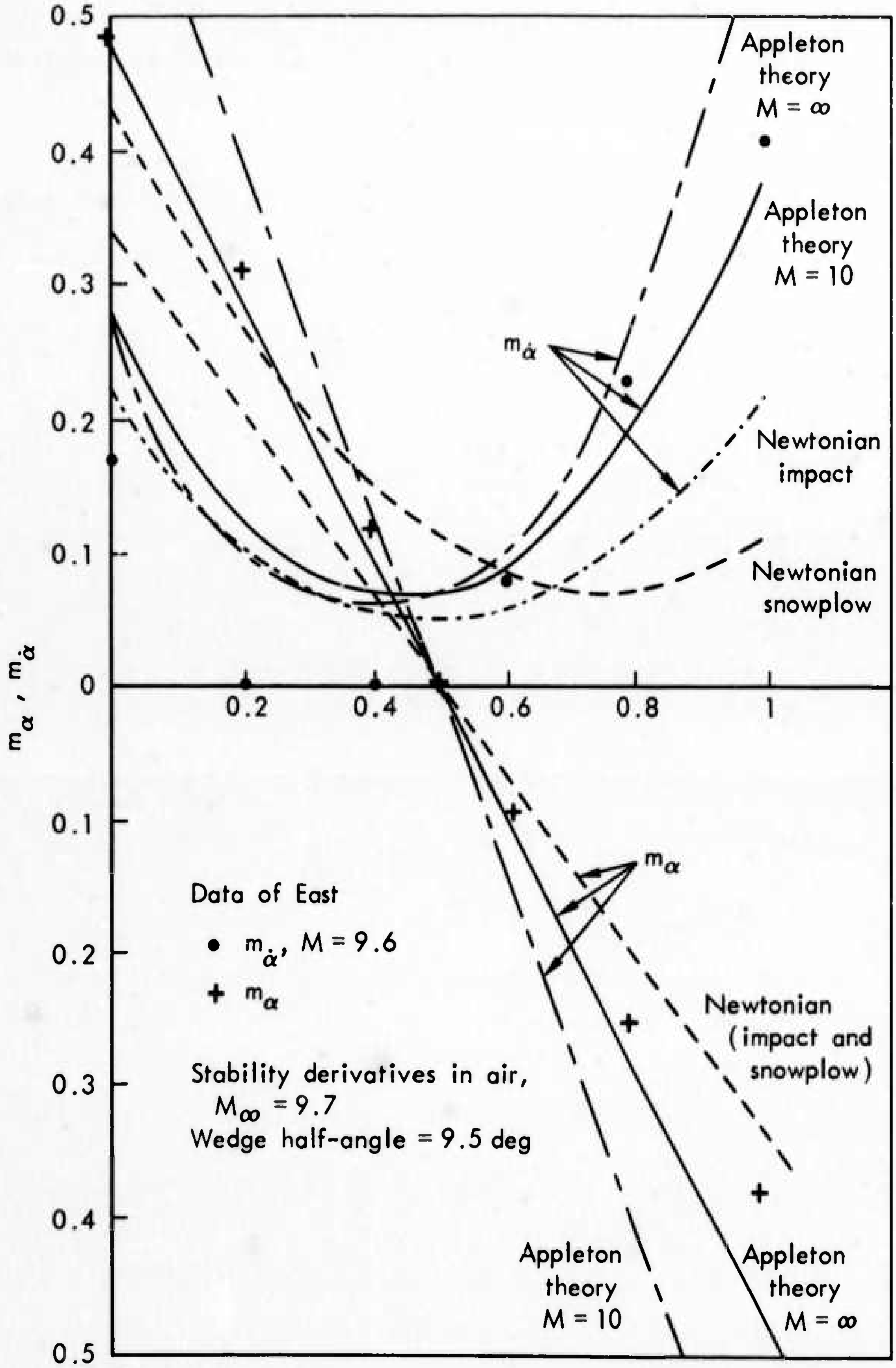

Fig.2—Aerodynamic stiffness and aerodynamic damping on slender wedges in high-speed flow

$$m_{\dot{\alpha}} = 4\left(\ell_1^2 - \ell_1 + \frac{1}{3}\right) \qquad (23)$$

While the present theory gives good results for the stiffness coefficient, m_{α}, it seems to be only qualitatively correct for the damping coefficient, $m_{\dot{\alpha}}$, and does not agree very well with the experimental data. It should be pointed out, however, that there were experimental difficulties, associated with side-wall interference effects, which may be reflected in the test data.

III. CONCLUSION

Newtonian theory is a systematic approximation to the equations of hypersonic gasdynamics for slender bodies in steady and unsteady flight. Although it is most accurate for cases where $\gamma \to 1$, in steady flow it leads to simple and useful approximations for pressure and force coefficients in air, where $\gamma = 1.4$.

In this Memorandum, Newtonian theory has been extended to the calculation of force and pressure coefficients on an airfoil undergoing oscillations about a mean translational motion, where the amplitude of the oscillations may be of the same order as the body thickness.

The theory is thus not restricted to small-angle oscillations; it should be applicable to both linear and nonlinear excursions from a mean steady flow.

The specific case of a slender wedge undergoing small-angle oscillations in air has been treated here, and stability derivatives have been compared to those which have already been determined theoretically using hypersonic small-disturbance theory, and experimentally in gun-tunnel studies at the University of Southampton.

The comparison indicates that Newtonian theory is not very accurate for the calculation of the out-of-phase component of pressure and pitching moment when $\gamma = 1.4$ and, in fact, seems to be no more accurate than simple impact theory. Neither impact theory nor Newtonian theory is capable of giving the out-of-phase component of the pressure with acceptable accuracy if $\gamma > 1$, although the in-phase component is given satisfactorily. Clearly then, a simple and accurate technique for engineering estimates of unsteady force coefficients has yet to be developed.

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10. ABSTRACT

Calculation of unsteady forces acting on two-dimensional oscillating airfoils by Newtonian snowplow theory. A pressure formula, valid for linear and nonlinear oscillations is obtained, and comparisons are made with the exact, linearized gas-dynamic theory for a hypersonic slender wedge. The comparisons indicate that, while Newtonian theory is less accurate than simple impact theory, neither gives reliable estimates of unsteady stability derivatives in air.

11. KEY WORDS

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Airfoils
Fluid dynamics
Gas dynamics
snowplow theory
Oscillation
Hypersonic flow
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